

COMBINATORICS AND GRAPH THEORY
PROFESSOR BREITENBUCHER
EXAM MIDTERM 2
PORTION TAKE HOME

NAME: _____
DATE: _____

I have neither given aid to nor received aid from a fellow student.

Signature: _____

Question 1.

(10 points) Find an explicit formula for the numbers a_n if $a_{n+1} = (n+1)a_n + 2(n+1)!$ if $n \geq 1$, and $a_0 = 0$.

Question 2.

(10 points) Prove that in any simple graph, there are two vertices with the same degree.

Question 3.

(10 points) Is there a simple graph G on seven vertices such that it is not connected, and each vertex of G has degree at least three?

Question 4.

(10 points) Show that the number of partitions of n is equal to the number of partitions of $2n$ into n parts.

Question 5.

(10 points) Use the PIE to find the number of solutions to the equation

$$x_1 + x_2 + x_3 = 16$$

where $x_1 \leq 3$, $x_2 \leq 4$, and $x_3 \leq 6$.