

MATH 111-01  
PROFESSOR BREITENBUCHER  
EXAM III, FORM: A

NAME: \_\_\_\_\_

STUDENT ID: \_\_\_\_\_

DATE: \_\_\_\_\_

There are **120 possible points**. The exam will be **graded out of 100 points**. Any points you receive above 100 will be counted as bonus points. You must show all work and justify all of your answers to receive full credit. You must work on the exam by yourself, to do otherwise is a violation of the Code of Academic Integrity. If you have questions about a problem, please ask for clarification. Good luck!

1. (15 points) Sketch the graph of a function satisfying the following

(a)  $f(1) = \frac{-1}{2}, f(3) = 2, f(-2) = 5, f(5) = \frac{1}{2}$

(b)  $f'(x) > 0$  for  $x < 0, 0 < x < 1,$  or  $x > 4$

(c)  $f'(x) < 0$  for  $1 < x < 2$  or  $2 < x < 4$

(d)  $f''(x) > 0$  for  $x < 0$  or  $2 < x < 6$

(e)  $f''(x) < 0$  for  $0 < x < 2$  or  $x > 6$

(f)  $x = 0$  and  $x = 2$  are vertical asymptotes and  $y = 1$  is a horizontal asymptote as  $x \rightarrow \infty$  and  $x \rightarrow -\infty$ .

2. Find the following

(a)  $\int \left( u^2 + 1 + \frac{1}{u^2} \right) du$  (5 points)

(b) the derivative of  $F(x) = \int_2^{x^2} \frac{\tan(t)}{t^2} dt$  (5 points)

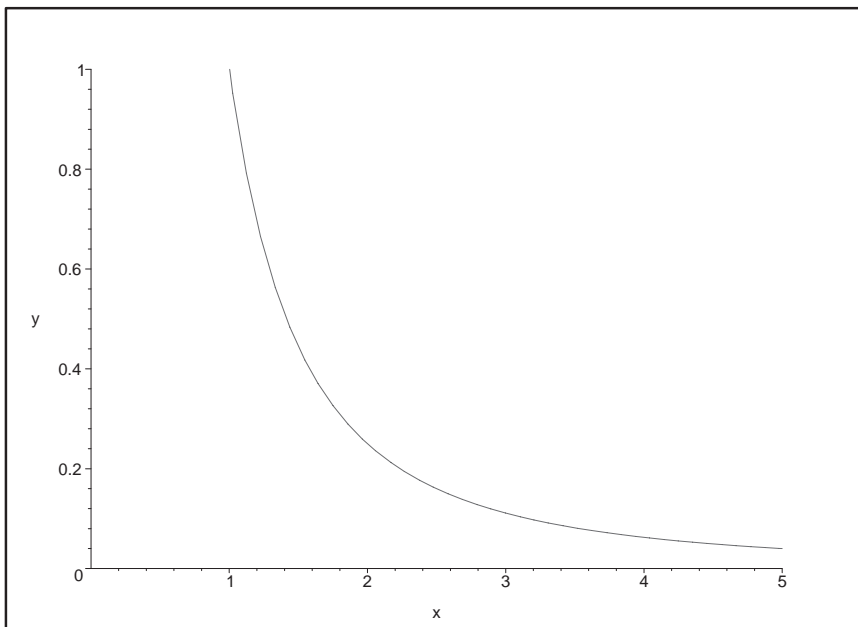
(c)  $\int (1 - t)(2 + t^2) dt$  (5 points)

(d) the derivative of  $h(x) = \int_1^2 (1 + y^2) dy$  (5 points)

3. The graph of  $y = \frac{1}{x^2}$  is given below. We want to estimate  $\int_1^5 \frac{1}{x^2} dx$ .

(a) Find  $S_4$  for the partition  $P = \{1, 1.6, 2.4, 3.6, 5\}$ , using  $x_1^* = 1, x_2^* = 2, x_3^* = 3, x_4^* = 5$ . (4 points)

(b) On the graph below indicate what area is represented by  $S_4$  in (a). (6 points)



4. A Norman window has the shape of a rectangle with a semicircle on top. If the perimeter of the window is 30 ft., find the dimensions of the window which allows the most light to enter. (10 points)

5. (a) State the Fundamental Theorem of Calculus precisely. (7 points)

(b) In your own words, tell me what the Fundamental Theorem of Calculus is saying. (3 points)

6. (a) The iteration scheme for Newton's Method is given by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

What does Newton's Method give you and where does it come from? (Be as detailed as possible. You may want to use a picture to help describe your ideas.) (9 points)

- (b) Use Newton's Method with  $f(x) = x^9 - 2$  and  $x_0 = 1$  to find  $x_1$  and  $x_2$ . (6 points)

7. You spot a deer in the road 550 ft. ahead. You are traveling at 68 mph (100 ft./s). You have good brakes and can decelerate at  $10 \text{ ft/s}^2$ . Will you be able to avoid hitting the deer? (15 points)

8. Find the area under the curve  $y = x^5$ , that is above the  $x$ -axis and between  $x = 1$  and  $x = 3$ . (10 points)

9. A manufacturer has been selling 1000 television sets a week at \$450 each. A marketing survey indicates that for each \$10 rebate offered to consumers, the number of sets sold will increase by 100 per week.

(a) Find the demand (price) function in terms of  $q$ , the number of sets sold. (3 points)

(b) How large a rebate should be offered to maximize revenue? (6 points)

(c) If weekly cost is  $C(q) = 68000 + 150q$ , what rebate should be offered to maximize profit? ( $q$  is the number of sets) (6 points)